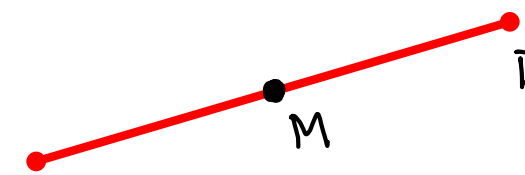
Math 115
Winter 2017
Lecture 7

Line Segment from A to B



$\overline{AB}$

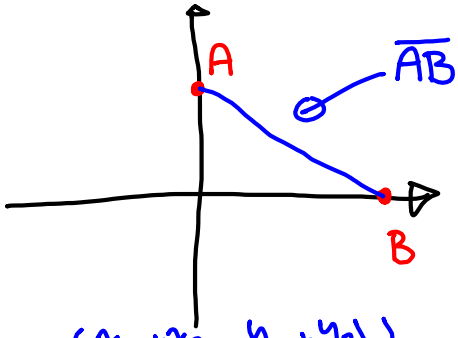
Distance between them $d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Midpoint $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope $m = \frac{y_1-y_2}{x_1-x_2}$ or $m = \frac{y_2-y_1}{x_2-x_1}$

Consider the line segment A B where

A(0,6) and B(8,0)



$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{0+8}{2}, \frac{6+0}{2}\right)$$

$$\boxed{M(4,3)}$$

$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$= \sqrt{(0-8)^2+(6-0)^2}$$

$$= \sqrt{(-8)^2+(6)^2}$$

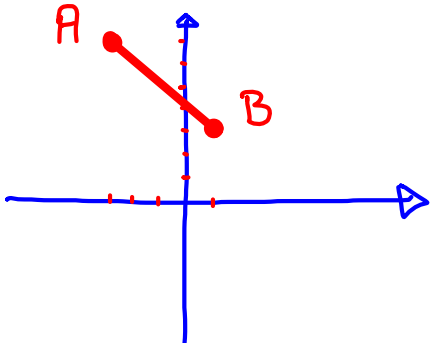
$$= \sqrt{64+36} = \sqrt{100} = \boxed{10}$$

Slope

$$m = \frac{y_1-y_2}{x_1-x_2} = \frac{0-6}{8-0} = \boxed{\frac{-3}{4}}$$

A(-3,7) , B(1,3)

① Draw $\overline{AB}$



② $$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$= \sqrt{(-3-1)^2+(7-3)^2}$$

$$= \sqrt{(-4)^2+(4)^2}$$

$$= \sqrt{16+16} = \sqrt{32} \approx \boxed{5.65}$$

③ $$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

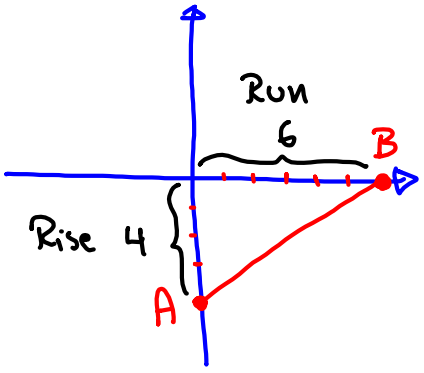
$$= M\left(\frac{-3+1}{2}, \frac{7+3}{2}\right)$$

$$= M\left(\frac{-2}{2}, \frac{10}{2}\right) = \boxed{M(-1,5)}$$

④ $$m = \frac{y_1-y_2}{x_1-x_2}$$

$$= \frac{7-3}{-3-1} = \frac{4}{-4} = \boxed{-1}$$

Repeat last example for $A(0,-4)$ & $B(6,0)$



$$m = \frac{\text{Rise}}{\text{Run}} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(0-6)^2 + (-4-0)^2}$$

$$= \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16}$$

$$= \sqrt{52} \approx \boxed{7.21}$$

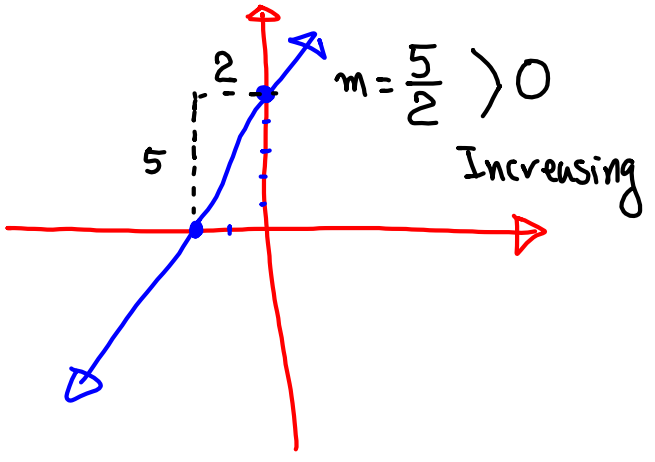
$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= M\left(\frac{0+6}{2}, \frac{-4+0}{2}\right)$$

$$= M\left(\frac{6}{2}, \frac{-4}{2}\right) = \boxed{M(3,-2)}$$

Graph using intercepts: $5x - 2y = -10$

x	y
0	5
-2	0



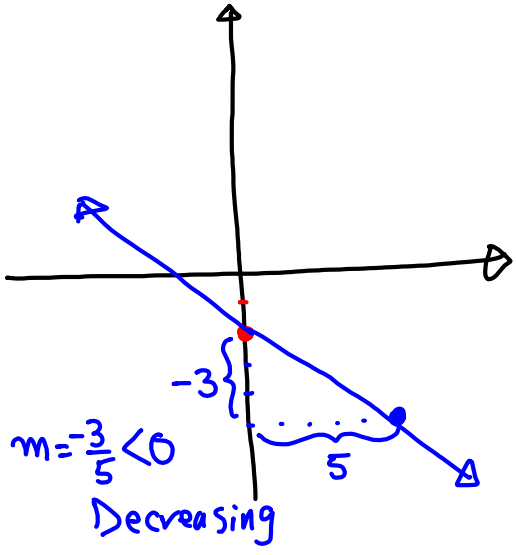
Convert to Slope-Int form,
Graph using slope & Int.

$3x + 5y = -10$

$5y = -3x - 10$

$y = \frac{-3}{5}x - 2$

$m = \frac{-3}{5}$, Y-Int $(0,-2)$



Graph using Point-slope

$y - 3 = \frac{1}{2}(x - 4)$

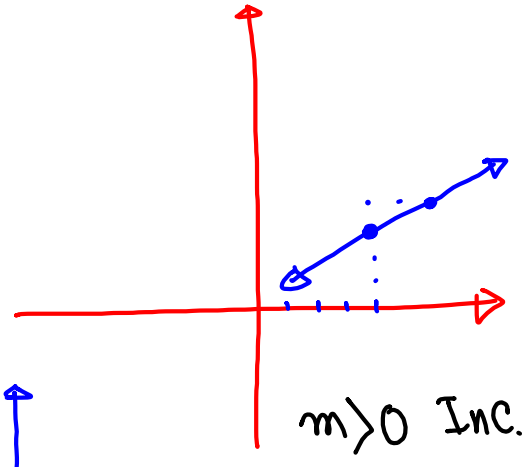
$m = \frac{1}{2}$

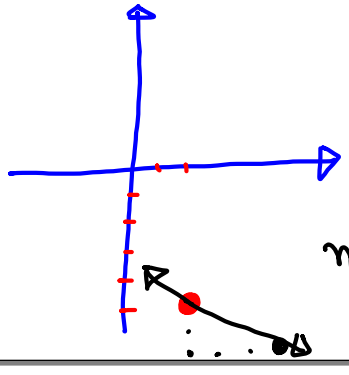
Point $(4,3)$

$y + 5 = \frac{-2}{3}(x - 2)$

$m = \frac{-2}{3}$

Point $(2,-5)$



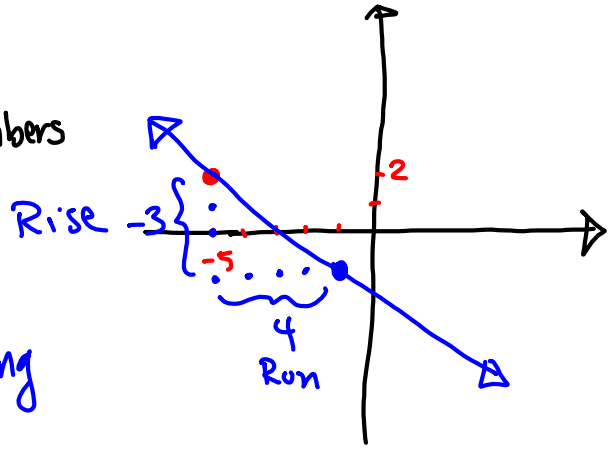


Graph: $y-2=\frac{-3}{4}(x+5)$

Slope $m=\frac{-3}{4}$

Point $(-5,2)$

Take opposite of numbers after x & y.



$m<0 \Rightarrow$ Decreasing

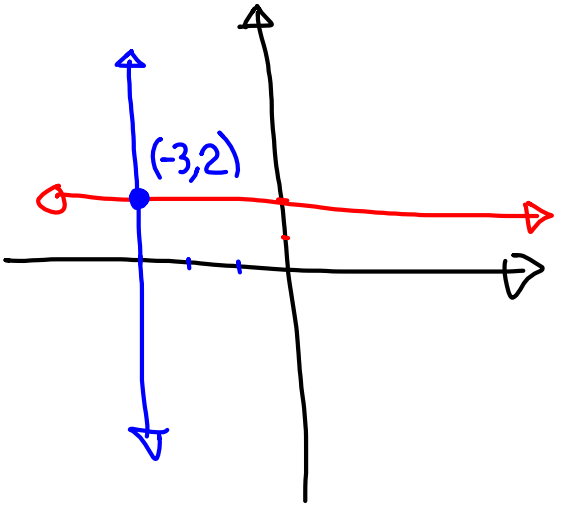
Graph

$y-2=0$ & $x+3=0$

$y=2$ $x=-3$

Y-only x-only

H.L. V.L.



Graph

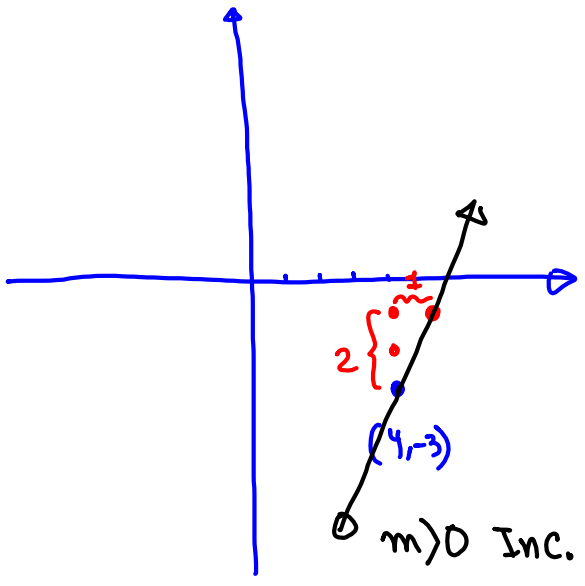
$y+3=2(x-4)$

Point – Slope form

$y-y_1=m(x-x_1)$

$m=2=\frac{2}{1}$ Rise / Run

Point $(4,-3)$



Write in Slope-Int form $\rightarrow y=mx+b$

$y+5=\frac{-2}{3}(x-4)$

① Distribute or multiply by LCD to remove fractions

LCD = 3

$3y+3\cdot5=\cancel{3}\cdot\frac{-2}{\cancel{3}}(x-4) \Rightarrow 3y+15=-2(x-4)$

$3y+15=-2x+8$

② Isolate Y

$3y=-2x+8-15$

$3y=-2x-7$

$y=\frac{-2}{3}x-\frac{7}{3}$

$m=\frac{-2}{3}$, Y-Int $(0,\frac{-7}{3})$

See last example to write

$y - 4 = \frac{2}{5}(x+3)$ in slope-Int form.

Distribute or use LCD to clear fractions.

LCD = 5

$5y - 20 = 2(x+3)$

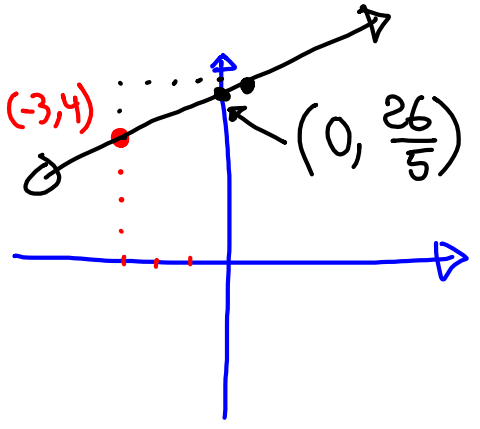
$5y - 20 = 2x + 6$

Isolate y

$5y = 2x + 26$

$\boxed{y = \frac{2}{5}x + \frac{26}{5}}$

$m = \frac{2}{5}$, Y-Int $(0, \frac{26}{5})$



Find eqn of a line that contains $(-3, 5)$ with

a) Zero slope

Horizontal line

Y-only

$y = 5$

b) No slope

Vertical line

X-only

$x = -3$

Find eqn of a line that contains $(4, -2)$ with

Hint: Use $y - y_1 = m(x - x_1)$

a) Slope 3

$y - y_1 = m(x - x_1)$

$y - (-2) = 3(x - 4)$

$y + 2 = 3x - 12$

$\boxed{y = 3x - 14}$

y-Int $(0, -14)$

b) Slope $\frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$y - (-2) = \frac{-1}{2}(x - 4)$

$y + 2 = \frac{-1}{2}(x - 4)$

$y + 2 = \frac{-1}{2}x + 2$

$\boxed{y = \frac{-1}{2}x}$

x-Int $(0, 0)$

Find eqn of a line that contains $(2, -3)$ & $(0, 4)$. Slope not given.

$(2, -3)$ & $(0, 4)$

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - 4}{2 - 0} = \boxed{\frac{-7}{2}}$$

Now use point-slope formula

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-7}{2}(x - 0)$

$y - 4 = \frac{-7}{2}x$

$\boxed{y = \frac{-7}{2}x + 4}$

See last example to find eqn of a line that contains $(-2,5)$ and $(3,1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-2-3} = \frac{4}{-5} = \boxed{-\frac{4}{5}}$$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{4}{5}(x - 3)$

$5y - 5 = -4(x-3)$

$5y - 5 = -4x + 12$

$5y = -4x + 17$

$$\boxed{y = -\frac{4}{5}x + \frac{17}{5}}$$

Find eqn of a line that contains the origin and is parallel to the line $2x - 3y = 6$.

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{2}{3}(x - 0)$

$$\boxed{y = \frac{2}{3}x}$$

Parallel lines $\Rightarrow$ Same Slope

$-3y = -2x + 6$

$y = \frac{-2}{-3}x + \frac{6}{-3}$

$y = \frac{2}{3}x - 2$

See last example to find eqn of a line that contains $(-4, 3)$ and is parallel to $2x+5y=8$.

$5y = -2x + 8$

$y = -\frac{2}{5}x + \frac{8}{5}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{2}{5}(x - (-4))$

$y - 3 = -\frac{2}{5}(x + 4)$

$5y - 15 = -2(x+4)$

$5y - 15 = -2x - 8$

$5y = -2x + 7$

$\boxed{y = -\frac{2}{5}x + \frac{7}{5}}$

Find eqn of a line that contains $(0, -4)$ and is perpendicular to the line $y = \frac{2}{3}x - 1$.

opposite Reciprocal

$y - y_1 = m(x - x_1)$

$m = -\frac{3}{2}$

$y - (-4) = -\frac{3}{2}(x - 0)$

$y + 4 = -\frac{3}{2}x \Rightarrow \boxed{y = -\frac{3}{2}x - 4}$

See last example to find eqn of a line that contains $(3, -2)$ and is perpendicular to the line $3x - 2y = 8$. $\Rightarrow -2y = -3x + 8$

$y = \frac{3}{2}x - 4$

$y - y_1 = m(x - x_1)$

$y - (-2) = -\frac{2}{3}(x - 3)$

opposite Reciprocal

$-\frac{2}{3}$

$-\frac{2}{3} \cdot (-3)$

$y + 2 = -\frac{2}{3}x + 2$

$y = -\frac{2}{3}x$

SG 8, SG 9, SG 10

WP 6

Due Tuesday

Graphing inequality in two variables

$3x + 2y > 6$

① Write in slope-int form

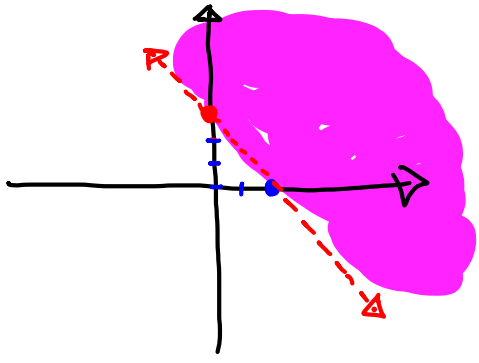
$2y > -3x + 6$

$y > \frac{-3}{2}x + 3$

Draw a broken line

Shade below when $<$

Shade above when $>$



Graph & shade:

$4x - 3y \geq 6$

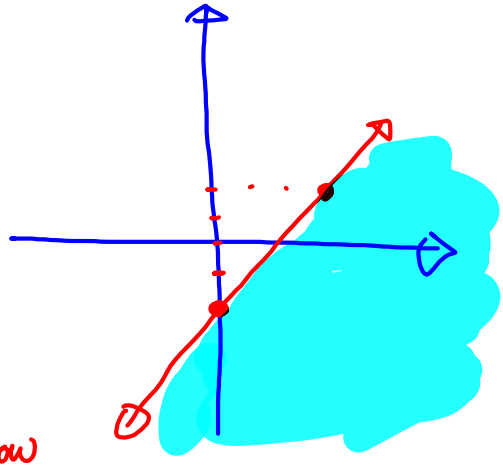
Rewrite in slope-int form

$-3y \geq -4x + 6$

$y \leq \frac{4}{3}x - 2$

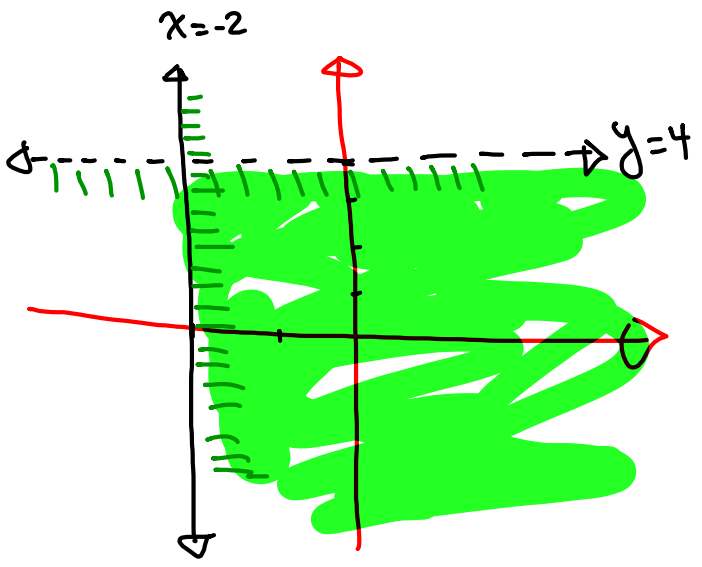
Solid line

Shade below



Graph & Shade

$$\begin{cases} x \geq -2 \\ y < 4 \end{cases}$$



Graph & Shade

$$\begin{cases} y \leq 3 \\ x > -2 \\ y \geq \frac{3}{4}x - 3 \end{cases}$$